GCE

Mathematics B MEI

H640/01: Pure Mathematics and Mechanics

A Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range. NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

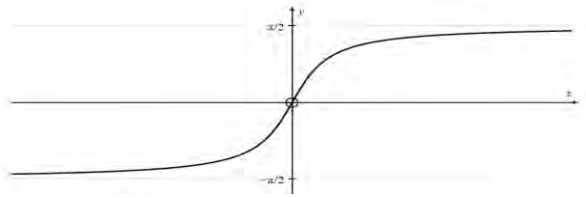
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

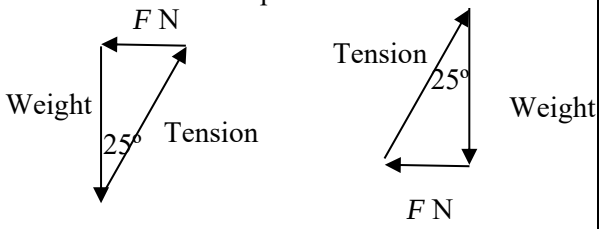
Question			Answer	Marks	AO	Guidance
1	(a)		Distance 2 + 5 + 4 = 11 m	M1 A1 [2]	1.1a 1.1b	adding distances in two or three sections cao
1	(b)		velocity = $\frac{5-1}{10-15}$ $v = -0.8$ m s^{-1}	M1 A1 [2]	1.1a 1.1b	Allow sign errors but not wrong way up. Soi Oe. Mark final answer`

Question			Answer	Marks	AO	Guidance	
2			$\frac{13-x}{(x-3)(x+2)} = \frac{A}{(x-3)} + \frac{B}{(x+2)}$ $13-x = A(x+2)+B(x-3)$ $A = 2,\ B = -3$ So $\frac{2}{(x-3)} - \frac{3}{(x+2)}$	M1 A1 A1 [3]	1.1a 1.1b 1.1b	Clearing the denominators oe For one correct coefficient Correct partial fractions seen Accept just values for A and B if defined	

Question			Answer	Marks	AO	Guidance	
3	(a)			B1 B1 [2]	1.1b 1.1b	General shape with horizontal asymptotes Allow if asymptote not drawn provided the intention is clear Must be a one-to-one function y-values $\pm\frac{\pi}{2}$ seen	
3	(b)		**DR** Graphs intersect when $3\sin x\cos x=\cos^2 x$ Either $\cos x=0$ giving $x=-\frac{\pi}{2},\frac{\pi}{2}$ or $3\sin x=\cos x$ giving $\tan x=\frac{1}{3}$ $x=0.322$, $x=-2.82$ to 3s.f. When $x=0.322$ or $x=-2.82$ $y=0.9$ [So the points of intersection are $(0.322,0.9),(-2.82,0.9)\left(-\frac{\pi}{2},0\right),\left(\frac{\pi}{2},0\right)$]	M1 M1 A1 M1 A1 A1 [6]	1.1a 1.1b 2.1 2.1 2.1 2.1	soi Attempt to solve $\cos x=0$ Both values in radians needed Both values in radians to at least 2 s.f. needed. Do not award if additional values inside the interval $[-\pi,\pi]$ Ignore additional values outside the interval $[-\pi,\pi]$. Allow awrt 0.90	Allow for $x=\tan^{-1}\frac{1}{3}$ SC1 award for 18.4° and -161.6° if 90° already seen Notice 0.9 is exact.

			Alternative method **DR**				
			Graphs intersect when $3\sin x\cos x = \cos^2 x$	**M1**		soi	
			Either $\cos x = 0$	**M1**	**1.1b**	Attempt to solve $\cos x = 0$	
			giving $x = -\frac{\pi}{2}, \frac{\pi}{2}$	**A1**	**2.1**	Both values in radians needed	
			Or $3\sin x = \cos x$ Squaring gives $9\sin^2 x = \cos^2 x = 1 - \sin^2 x$ $10\sin^2 x = 1$				
			$\sin x = \pm\sqrt{0.1}$ $x = -2.820, -0.322, 0.322, 2.820$	**M1**		Complete method for finding at least one value for $\sin x$	
			Select genuine roots 0.322, -2.820	**A1**		Both correct roots and no others in the range	
			When $x = 0.322$ or $x = -2.82$ $y = 0.9$	**A1**	**2.1**	Allow awrt 0.90	Notice 0.9 is exact.
			[So the points of intersection are $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0.322, 0.9), (-2.820, 0.9)$]				
				[6]			

Question			Answer	Marks	AO	Guidance	
4			**AG** $\frac{1}{(2+x)^2} = \frac{1}{4\left(1+\frac{x}{2}\right)^2} = \left[\frac{1}{4}\left(1+\frac{x}{2}\right)^{-2}\right]$	**B1**	**2.1**	Dealing correctly with the 2. Need not use negative powers for this mark	Note there is mathematically nothing wrong with the direct expansion $(2+x)^{-2} =$ $2^{-2} - 2\times 2^{-3}x + 3\times 2^{-4}x^2$ Award B1M1 if seen
			$= \frac{1}{4}\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\ldots\right)$	**M1**	**2.1**	Allow for expanding $(1+kx)^{-2}$ even where the B mark is not awarded	
			$\frac{1-x}{(2+x)^2} \approx \frac{(1-x)}{4}\left(1-x+\frac{3}{4}x^2\right)$	**M1**	**2.1**	Attempt to multiply their expansion by the numerator	
			$\approx \frac{1}{4} - \frac{1}{2}x + \frac{7}{16}x^2$	**A1**	**2.1**	Convincing argument	
			Alternative method $\frac{1}{(2+x)^2} = \frac{3}{(2+x)^2} - \frac{1}{2+x}$	**M1**		Using partial fractions – allow an arithmetic slip	
			$\frac{3}{(2+x)^2} = \frac{3}{4}\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\ldots\right)$	**B1**		Dealing correctly with the 2. Need not use negative powers for this mark	
			$-\frac{1}{2+x} = -\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}\ldots\right)$	**M1**		Allow for expanding both $(1+kx)^{-2}$ and $(1+kx)^{-1}$ even where the B mark is not awarded	
			$\approx \frac{1}{4} - \frac{1}{2}x + \frac{7}{16}x^2$	**A1**		Adding terms to complete a convincing argument	
				[4]			

Question			Answer	Marks	AO	Guidance
5	(a)		Either of these is acceptable  	B1 B1 [2]	1.1b 1.1b	Arrows making a closed loop in roughly the right directions Tension, weight and F labelled on their triangle and 25º (or 65°) correctly labelled. May be given as a suitable angle outside the triangle.
5	(b)		Using the triangle of forces $F = 3g\tan 25°$ or $\text{Tension} = \dfrac{3g}{\cos 25°}$ $F = 13.7$ $\text{Tension} = 32.4$ N **Alternative method** Resolve vertically $T\cos 25° = 3g$ $T = 32.4$ N Resolve horizontally $F = T\sin 25° = 13.7$	M1 A1 A1 M1 A1 A1 [3]	1.1a 1.1b 1.1b 	Allow sin/cos or 25°/65° interchange to find F or T cao cao Allow sin/cos interchange or 25°/65° interchange cao cao

Question			Answer	Marks	AO	Guidance	
6	(a)		• the bricks have negligible size so contact force with the plank acts at a point • The mass of plank is evenly spread across its length • the weight of plank acts at centre of plank.	B1 B1 [2]	3.3 3.3	Allow "no size" or "size doesn't matter" or "shape is not relevant" etc Allow for either statement Allow the plank is the same throughout or centre of mass at centre	Allow "weight of bricks acts on the plank at point" Do not allow "mass acts" at the centre
6	(b)		when placed at the centre tensions are equal $2\times 75 = 2.3ng + 5g$ $n = \left[\frac{101}{2.3g}\right] = 4.48...$ so 4 bricks Alternative using moments $5g\times 0.4 + 2.3gn\times 0.4 = 75\times 0.8$ $n = \left[\frac{40.4}{9.016}\right] = 4.48...$ so 4 bricks	M1 A1 M1 A1 [2]	3.1b 1.1b 1.1b	Using symmetry to establish equation for n soi. Allow if the weight of the plank or one of the tensions is missing Final answer must be the integer Allow if 4 seen www Allow for missing moment of weight or one of the tensions Final answer must be the integer Allow if 4 seen www	Trial and improvement may be used Every term must be a moment
6	(c)		$2.3gnx$ Nm	B1 B1 [2]	1.1b 1.2	Allow positive or negative $22.54nx$	Allow in an equation.
6	(d)		4 bricks on the point of breaking x m from A Taking moments about A $5g\times 0.4 + 4\times 2.3gx = 75\times 0.8$ $9.2gx = 60 - 2g$ $x = 0.448$ [so 44.8 cm from A]	M1 A1 A1	3.1b 1.1b 1.1b	Taking moments about any point to form an equation. FT their n All forces used in a moment. Allow sign errors. Allow an incorrect distance used. Could be an inequality Fully correct equation FT their n. Allow corresponding inequality Need not be simplified cao	

Question			Answer	Marks	AO	Guidance	
				[3]			

Question			Answer	Marks	AO	Guidance	
7	(a)		$x = 14t$	B1	1.1b	must be $x = \ldots$ or seen as the first component of the vector. Do not award for an expression that adds a vector to a scalar	SC1 for $\begin{pmatrix} 7t - 4.9t^2 + 5 \\ 14t \end{pmatrix}$ or $\begin{pmatrix} 14t - 4.9t^2 + 5 \\ 7t \end{pmatrix}$
			$y = 7t - \frac{1}{2}gt^2 + 5$	M1	1.1b	allow without +5, or if -5 seen Do not award for an expression that adds a vector to a scalar	
			So the position vector is $\begin{pmatrix} 14t \\ 7t - 4.9t^2 + 5 \end{pmatrix}$	A1	2.5	Must be a single vector. Accept $\frac{1}{2}g$ in final answer Accept $14t\mathbf{i} + \left(7t - 4.9t^2 + 5\right)\mathbf{j}$	
				[3]			
7	(b)		Lands when $y = 0$ $7t - 4.9t^2 + 5 = 0$	M1	3.1b	Award for correct quadratic or an attempt to find value of t when their quadratic $y = 0$	
			$t = 1.95$	A1	1.1b	cao	
			gives $x = 14t = 27.3$ m	B1	1.1b	FT their t and their linear expression for x ISW where candidates find the distance from the point of projection	
				[3]			

Question			Answer	Marks	AO	Guidance
8	(a)		$\frac{dx}{dt} = 3t^2 - 8, \quad \frac{dy}{dt} = 2t$	M1*	1.1a	attempt to differentiate both parametric equations Only allow for a complete method for finding $\frac{dy}{dx}$ in terms of t using the cartesian equation of the curve
			$\frac{dy}{dx} = \frac{2t}{3t^2 - 8}$	M1 (dep)	1.1b	Combine their derivatives to find $\frac{dy}{dx}$ Do not allow for reciprocal
				A1	1.1b	cao
				[3]		
8	(b)		**AG** $t^3 - 8t = 8$ and $t^2 = 4$ gives $t = -2$	M1	3.1a	Attempt to establish the value of t at (8, 4). Allow for ± 2 or 2 stated Allow for $y = 4$ used in $\frac{dy}{dx} = \frac{2\sqrt{y}}{3y - 8}$ for the M mark only
			$\frac{dy}{dx} = \frac{2(-2)}{3(-2)^2 - 8} = -1$	E1	1.1b	**AG** the negativity must be clearly established from correct working
			Alternative When $\frac{dy}{dx} = \frac{2t}{3t^2 - 8} = -1$ giving $3t^2 + 2t - 8 = 0$	M1		Uses the value of the derivative to find the value of t at P.
			$t = \frac{4}{3}$ or $t = -2$			
			When $t = -2$ the coordinates are $\left((-2)^3 - 8(-2), (-2)^2\right) = (8, 4)$ [which is P]	E1		Allow without reference to $t = \frac{4}{3}$
				[2]		
8	(c)		$\frac{dy}{dx} = \frac{2t}{3t^2 - 8} = -1$ giving $3t^2 + 2t - 8 = 0$	M1	1.1a	Equating their $\frac{dy}{dx}$ to -1 and rearranging to form quadratic equation
			$t = \frac{4}{3}$ or [$t = -2$ is the point P]	A1	3.2a	allow www –2 need not be seen but if seen must be rejected
				[2]		

8	**(d)**		Substitute $t^2 = y$	**M1**	**1.1a**	
			$x = t^3 - 8t \Rightarrow x^2 = t^6 - 16t^4 + 64t^2$	**A1**	**1.1b**	Allow for $x^2 = t^2\left(t^2 - 8\right)^2$
			$\Rightarrow x^2 = y^3 - 16y^2 + 64y$	**A1**	**2.1**	
			Alternative method			
			Substitute $t = \pm y^{\frac{1}{2}}$	**M1**		Substituting for *t* in their equation for *x*; allow without $\pm$
			$x = \pm\left(y^{\frac{3}{2}} - 8y^{\frac{1}{2}}\right)$	**A1**		Soi Allow without $\pm$
			$x^2 = y(y-8)^2 = \left[y^3 - 16y^2 + 64y\right]$	**A1**		must be in the form $x^2 = \ldots$ from fully correct working Need not be simplified. Do not award if $\pm$ not seen at all
				[3]		

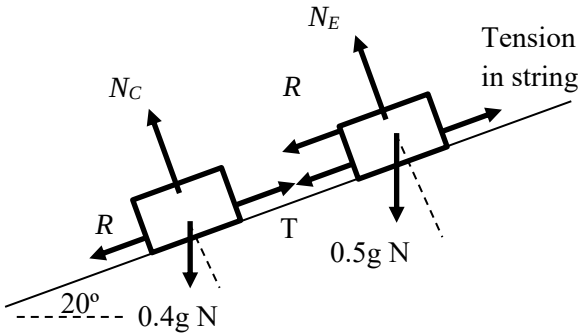
9	(a)		$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 2k\mathbf{i} + 6\mathbf{j}$ When $t = 2$, $\mathbf{a} = 2 \times 2k\mathbf{i} + 6\mathbf{j}$ $\lvert\mathbf{a}\rvert = \sqrt{(4k)^2 + 6^2} = 10$ giving $16k^2 + 36 = 100$ So $k = 2$	**M1** **M1** **M1** **A1** **[4]**	**3.1a** **1.1b** **3.1a** **3.2a**	differentiating the **v** vector substituting $t = 2$ into their **a** vector Equate the magnitude of their **a** vector to 10 must choose the positive value if two values seen
9	(b)		$\mathbf{r} = \int \mathbf{v}\, dt = \frac{kt^3}{3}\mathbf{i} + 3t^2\mathbf{j} + \mathbf{c}$ particle at the origin when $t = 0$ so $\mathbf{c} = \mathbf{0}$ So $\mathbf{r} = \frac{kt^3}{3}\mathbf{i} + 3t^2\mathbf{j} = \left[\frac{2t^3}{3}\mathbf{i} + 3t^2\mathbf{j}\right]$	**M1** **A1** **[2]**	**1.1a** **1.1b**	integrating with their k or general k. Allow for a vector or for both components separately integrated. Condone missing +**c** or +**c** still in their answer FT their k if positive or general k used Must be in vector form
9	(c)		Northeast when the **i** component = **j** component $\frac{2t^3}{3} = 3t^2$ giving $t = 4.5$ s [$t = 0$ rejected as the particle is at the origin]	**M1** **A1** **[2]**	**3.1b** **1.1b**	FT their **r** www

Question			Answer	Marks	AO	Guidance	
10			Length BC: $l^2 = 30^2 + 15^2 - 2\times 30\times 15\cos\theta$	M1	3.1a		If M0 awarded here allow SC1 for $BC = \sqrt{675} = 15\sqrt{3}$ found using $\theta = \frac{\pi}{3}$
			$l^2 = 1125 - 900\cos\theta$ $l = (1125 - 900\cos\theta)^{1/2}$	A1	1.1b	Soi Allow equivalent in metres	If working in metres $l^2 = 0.1125 - 0.0900\cos\theta$
			$\frac{dl}{d\theta} = \frac{1}{2}(1125 - 900\cos\theta)^{-\frac{1}{2}} \times 900\sin\theta$	M1 A1	3.1a 1.1b	Attempt to use the chain rule Any form	
			$\frac{d\theta}{dt} = 0.1$	B1	1.2	Soi eg from $\theta = 0.1t$	
			$\frac{dl}{dt} = \frac{dl}{d\theta} \times \frac{d\theta}{dt} = \frac{450\sin\theta}{(1125 - 900\cos\theta)^{\frac{1}{2}}} \times 0.1$	M1	1.1a	Using the chain rule to find $\frac{dl}{dt}$	
			When $\theta = \frac{\pi}{3}$ $\frac{dl}{dt} = \frac{45\sin\frac{\pi}{3}}{(1125 - 900\cos\frac{\pi}{3})^{\frac{1}{2}}} = \left[\frac{45\sqrt{3}}{2\times 15\sqrt{3}} = \frac{3}{2}\right]$	M1	1.1a	Substitute $\theta = \frac{\pi}{3}$ into their $\frac{dl}{d\theta}$	
			1.5 cm s^{-1}	A1 [8]	3.2a	Must have correct unit for the value Allow written as cm per second oe	0.015 m s^{-1}

Question			Answer	Marks	AO	Guidance	
			Alternative method				
			$l^2 = 30^2 + 15^2 - 2\times 30\times 15\cos\theta$	**M1**			
				A1			If working in metres $= 0.1125 - 0.0900\cos\theta$
			$l^2 = 1125 - 900\cos\theta$				
			$2l\dfrac{\mathrm{d}l}{\mathrm{d}\theta} = 900\sin\theta$	**M1** **A1**		Attempt to use the implicit differentiation. Any form	
			$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = 0.1$	**B1**		soi	
			$\dfrac{\mathrm{d}l}{\mathrm{d}t} = \dfrac{\mathrm{d}l}{\mathrm{d}\theta}\times\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{450\sin\theta}{l}\times 0.1$	**M1**		Using the chain rule to find $\dfrac{\mathrm{d}l}{\mathrm{d}t}$	
			When $\theta = \dfrac{\pi}{3}$ $\dfrac{\mathrm{d}l}{\mathrm{d}t} = \dfrac{45\sin\frac{\pi}{3}}{15\sqrt{3}} = \dfrac{3}{2}$	**M1**		Substitute $\theta = \dfrac{\pi}{3}$ into their $\dfrac{\mathrm{d}l}{\mathrm{d}\theta}$	
			1.5 cm s^{-1}	**A1**		Must have correct unit for the value Allow written as cm per second oe	0.015 m s^{-1}
				[8]			

Question			Answer	Marks	AO	Guidance	
11			Let $u = 2x + k$ $2\mathrm{d}x = \mathrm{d}u$	M1	2.1	Substituting $u = 2x + k$ Allow for a different substitution giving an integral in u only	Allow for $\int \frac{a}{u^2}\mathrm{d}u$ for any constant seen
			$\int \frac{2}{(2x+k)^2}\mathrm{d}x = \int \frac{1}{u^2}\mathrm{d}u$	A1	2.1	Correct integrand in terms of u Ignore limits	
			$= -\frac{1}{u}[+c]$	A1	2.1	correct indefinite integral constant need not be seen	
			$\int_k^{2k} \frac{2}{(2x+k)^2}\,\mathrm{d}x = \int_{3k}^{5k} \left(\frac{1}{u^2}\right)\mathrm{d}u = -\frac{1}{5k} + \frac{1}{3k}$	M1	2.1	substituting correct new limits into their integrated expression, or substituting in terms of x and using original limits	
			Alternatively, by inspection $\int_k^{2k} \frac{2}{(2x+k)^2}\,\mathrm{d}x = \left[-(2x+k)^{-1}\right]_k^{2k}$	M1 A2		Integrating by inspection to obtain any multiple of $(2x+k)^{-1}$ Fully correct indefinite integral – need not be simplified.	
			$-\frac{1}{5k} + \frac{1}{3k}$	M1		substituting limits into their integrated expression	
			$= \frac{2}{15k}$	A1	2.1	Allow $\left(-\frac{1}{5} + \frac{1}{3}\right)\frac{1}{k}$ seen	
			This is inversely proportional to k [with constant of proportionality $\frac{2}{15}$]	E1	2.2a	FT their definite integral Must use phrase "inversely proportional" to k or indicates $\propto \frac{1}{k}$	Allow if $\frac{a}{k}$ required at the start of the argument
				[6]			

Question			Answer	Marks	AO	Guidance
12			Assume there is a prime number p which is one less than a square number $p = n^2 - 1$ for some positive integer $n \geq 2$	**M1***	**2.1**	Setting up proof by contradiction
			$p = (n-1)(n+1)$	**M1***	**2.1**	factorising
			If $n = 2$ $p = 1 \times 3 = 3$ which is prime [$p = 2$ is not 1 less than a square number]	**E1**	**2.1**	Considers the possibility that one factor might be 1
			If $n > 2$ then p has two [proper] factors so is not prime which is a contradiction. So there are no prime numbers other than 3 which are 1 less than a square number	**E1 (dep)**	**2.1**	Condone missing reference to $n = 2$ (or $p = 3$) for this step. Conclusion must be clear. Allow SC1 where M1M0 or M0M0 has been awarded and $3 = 2^2 - 1$ is established
				[4]		

Question			Answer	Marks	AO	Guidance
13	(a)		Newton's second law for the train $5-2\times0.8=(0.5+0.4)a$	**M1**	**3.1b**	N2L for whole train with correct mass and all forces present
			Alternative $5-0.8-T=0.5a$ $T-0.8=0.4a$			Also allow for 2 equations where both have correct mass and all forces present in each
			giving $a=\frac{34}{9}=3.78\ \text{m s}^{-2}$.	**A1**	**1.1b**	
			Using $v=u+at$ with $u=0, t=1.5$	**M1**	**3.1b**	using *suvat* equation(s) with $u=0$ and their $a\neq g$ leading to a value for *v*
			$v=\frac{34}{9}\times1.5=\frac{17}{3}=5.67\ \text{m s}^{-1}$ (3sf)	**A1** **[4]**	**1.1b**	FT their *a*. Any form
13	(b)			**B1** **B1** **B1** **[3]**	**1.1b** **1.1b** **1.1b**	weights and normal reactions (must be distinct and not vertical) Allow if both components of weight given instead. Allow in addition to weight only if clear they are for working purposes only tensions in string and coupling parallel to inclined plane *R* marked for both parts of the train. No additional forces Allow if distinct if it is clear they are equal in later work
13	(c)		Newton's second law $P-2R-0.9g\sin20°=0.9a$	**M1** **A1** **[2]**	**1.1b** **3.3**	Newton's law with $m = 0.9$. Allow for incorrect weight term(s) or R used instead of 2R Fully correct Any form

Question			Answer	Marks	AO	Guidance
13	(d)		When $P=5$ the equation gives $5-2R-0.9g\sin 20°=0.9a$	M1	3.1b	establishes one equation linking *R* and *a*. FT their (c)
			When $P=5.5$ the equation gives $5.5-2R-0.9g\sin 20°=0.9\times 2a$	M1	3.1b	establishes another equation linking *R* and *a*. Consistent with their first equation
			Solve simultaneously giving $R=0.742$ $\left[a=\frac{5}{9}\right]$	A1	1.1b	method need not be seen BC correct value for *R* (*a* is not required)
			Alternative method When $P=5$ $a=\frac{5-2R-0.9g\sin 20°}{0.9}$ When $P=5.5$ $a_1=\frac{5.5-2R-0.9g\sin 20°}{0.9}$ So $\frac{5.5-2R-0.9g\sin 20°}{0.9}=2\left(\frac{5-2R-0.9g\sin 20°}{0.9}\right)$ giving $R=0.742$ $\left[a=\frac{5}{9}\right]$	M1 M1 A1		Finds expression for *a* when $P=5$ or $P=5.5$ Soi Links corresponding acceleration for the other value of *P* Do not allow factor of 2 on the wrong side correct value for *R* (*a* is not required)
				[3]		

Question			Answer	Marks	AO	Guidance
14	(a)		When $t = 0, \quad 82 = \theta_0 e^0$ so $\theta_0 = 82$ $t = 5, \quad 27 = \theta_0 e^{-5k}$ giving $k = \left[-\frac{1}{5}\ln\left(\frac{27}{82}\right)\right] = 0.222$ to 3 sf	B1 M1 A1 [3]	3.3 3.3 1.1b	 Forming an equation for *k* and attempt to solve Allow for exact value or evaluated to at least 2 s.f.
14	(b)		The model predicts that temperature tends to zero but if the quantity of water is small the water will warm up so it will not cool the object to zero.	E1 [1]	3.5b	Must imply to the model tends to zero and this does not match the real situation.
14	(c)		$\ln\theta = \ln\left(\theta_0 e^{-kt}\right) = \ln\theta_0 + \ln\left(e^{-kt}\right)$ $\ln\theta = \ln 82 - 0.222t = \left[4.41 - 0.222t\right]$	M1 A1 [2]	2.1 2.1	Taking logs and attempting to use laws of logs Do not award for values of *a* and *b* obtained directly from the data and the natural log form of the model. FT their values for θ_0 and *k* Accept as part of equation or *a* and *b* clearly stated
14	(d)		When $t = 0, \ln\theta = 3.4$ giving $\theta = 29.96$ so 30.0° C to 3 sf $\theta = 29.96e^{-0.08t}$ $\frac{d\theta}{dt} = 29.96 \times -0.08e^{-0.08t}$ When $t = 0 \quad \frac{d\theta}{dt} = -2.3968$ [object is cooling by 2.4°per minute]	B1 M1 A1 A1 [4]	3.4 3.4 3.4 3.4	Accept 30° www Must be evaluated Attempt to differentiate their exponential expression for θ Any form eg $e^{3.4} \times -0.08e^{-0.08t}$ or $-0.08e^{3.4-0.08t}$ Allow for correct negative value for $\frac{d\theta}{dt}$ or a clear statement that the rate of cooling is 2.4° per minute. Accept $= -0.08e^{3.4}$

			Alternative method When $t = 0, \ln\theta = 3.4$ giving $\theta = 29.96$ so 30.0° C to 3 sf Differentiate $\ln\theta = 3.4 - 0.08t$ w.r.t t $\frac{1}{\theta}\frac{d\theta}{dt} = -0.08$ $\frac{d\theta}{dt} = -0.08\theta$ When $t = 0, \theta = 29.96$ so $\frac{d\theta}{dt} = -2.3968$ object is cooling by 2.4°per minute	**B1** **M1** **A1** **A1** **[4]**	**3.4**	Accept 30° www Uses implicit differentiation w.r.t t Correct derivative Allow for correct negative value for $\frac{d\theta}{dt}$ or a clear statement that the rate of cooling is 2.4°per minute	
14	**(e)**		Solve simultaneously $\ln\theta = 3.4 - 0.08t$ $\ln\theta = \ln 82 - 0.222t$ gives $t = 7.089$ $t = 7.1$ [7 minutes and 5 seconds] $\ln\theta = 2.8328$ gives $\theta =$ 17° C Alternative method $82e^{-0.222t} = 30e^{-0.08t}$ $\frac{82}{30} = e^{0.142t}$ $t = 7.08$ [7 minutes and 5 seconds] $\theta = 17$ ° C	**M1** **A1** **A1** **M1** **A1** **A1** **[3]**	**3.1b** **3.4** **3.4**	Attempting to find the intersection of their (c) and the given line Accept awrt 7.0, 7.1 or 7.2 Must be the value for θ Equate their expressions for temperature and attempts to solve for t Accept awrt 7.0, 7.1 or 7.2 Cao	Could be BC

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